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A METHOD FOR PREVIEW VIBRATION CONTROL OF SYSTEMS HAVING FORCING INPUTS AND RAPIDLY-SWITCHED DAMPERS

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In a variety of applications, especially in large scale dynamic systems, the mechanization of different vibration control elements in different locations would be decided by limitations placed on the modal vibration of the system and the inherent dynamic coupling between its modes. Also, the quality of vibration control to the economy of producing the whole system would be another trade-off leading to a mix of passive, active and semi-active vibration control elements in one system. This term *active* is limited to externally powered vibration control inputs and the term *semi-active* is limited to rapidly switched dampers. In this article, an optimal preview control method is developed for application to dynamic systems having active and semi-active vibration control elements mechanized at different locations in one system. The system is then a piecewise (bilinear) controller in which two independent sets of control inputs appear additively and multiplicatively. Calculus of variations along with the Hamiltonian approach are employed for the derivation of this method. In essence, it requires the active elements to be ideal force generators and the switched dampers to have the property of on-line variation of the damping characteristics to pre-determined limits. As the dampers switch during operation the whole system's structure differs, and then values of the active forcing inputs are adapted to match these rapid changes. Strictly speaking, each rapidly switched damper has pre-known upper and lower damping levels and it can take on any in-between value. This in-between value is to be determined by the method as long as the damper tracks a pre-known fully active control demand. In every damping state of each semi-active damper the method provides the optimal matching values of the active forcing inputs. The method is shown to have the feature of solving simple standard matrix equations to obtain closed form solutions. A comprehensive 9-DOF tractor semi-trailer model is used to demonstrate the effectiveness of the method. Time domain predictions are made to compare performance of ride and tyre-to-road contact in the model for the presented method with those of some other active and semi-active suspension designs.

1. INTRODUCTION

Manufacturers have begun seriously to apply active and semi-active suspensions because of their effective control of vibrations in many industrial applications. Doubtless, the automobile industry is one of the most important areas of application of these suspensions, but the term suspension cannot be confined in any way to the vehicle industry because so many engineering applications are using suspensions. Active suspensions can play a very vital role in controlling vehicle vibrations to limits unreachable by any other kind of suspensions [1–3]. On the other hand, semi-active suspensions can be a good compromise between high performance capabilities with complexity for active suspensions and low performance capabilities with simplicity for passive suspensions.

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The topic of active and semi-active suspensions has been subject to extensive studies by many researchers in the past three decades [1]. An active suspension controller requires a fast actuation of vibration control force by using an external power supply, while semi-active control strategies are based on simpler signal processing and a very low amount of power supply as compared to active ones [4, 5]. Optimization of active suspensions by applying methods of modern control theory has been reported in references [2, 3]. Semi-active suspensions are strongly non-linear systems due to rapid switching of damping from one level to another in order to operate continuously the semi-active damper in an energy dissipation mode. This job is often done by varying the cross-sectional area of the oil flow of these dampers. Therefore, performance optimization of them is not an easy task, but many attempts have been made [6–8].

The idea of having pre-knowledge of road roughness by using sensors installed in the front of a vehicle is known as preview (action) control. This idea has been originally developed by Bender [9]. Preview active suspensions are capable of providing performance superior to that of all the other types of modern suspensions [10]. In addition, they can significantly resolve the inevitable conflict between the many performance requirements of vehicle suspensions [11]. Optimal semi-active control policies with preview have been developed for application to vehicle suspensions on simple and complicated models [13–16]. In these publications, the authors have shown that the preview action improves, to a great extent, the performance measures of semi-active suspensions.

In this paper, a method for preview vibration control is presented for application to systems having both active (forcing) and semi-active rapidly-switched elements installed at different positions. Any combination of passive, active and semi-active isolation elements could exist in one system. Since the passive elements have constant parameters, the method developed here is to find optimum values of the active forces and the semi-active damping rates to satisfy the requirements of a generalized quadratic performance index. The method also gives their optimum values with/without the preview action. The model used for application is a 9-DOF commercial vehicle.

2. SYSTEM MODEL

The model considered in this study is shown in Figure 1. It is a 9-DOF tractor semi-trailer vehicle. Obviously, it is a two-dimensional model in the vertical and longitudinal planes. The detailed mathematical description of this model and its equations of motion have been reported in reference [17], for the case in which can be regarded as a linear lumped parameter model. Also, values of the vehicle parameters have been tabulated in references [17–19]. Many (assumptions) degrees of idealization are made in order to determine the vehicle degrees of freedom; the most important ones are (1) it travels at constant speed on an uneven road, (2) the tractor, the cab and the semi-trailer are perfectly rigid, (3) the semi-trailer and the cab are allowed to make translational motion, (4) the passive spring and damper elements are linear, and (5) the forcing inputs are ideal, i.e. the actuator nonlinearities are ignored. These assumptions lead to a 9-DOF linear vibratory motion which includes bouncing and pitching motions of both the cab and the tractor centres of gravity, y_c, θ_c, y_t and θ_t, pitching motion of the semitrailer centre of gravity, θ_S, and the bouncing motions of each wheel-axle assembly, y_1, y_2, y_3 and y_4. Note here that x_c, x_s and x_t are three dependent longitudinal motions of the cab, the semi-trailer and the tractor, respectively. The dimensions of the vehicle in the longitudinal direction are denoted by $b_1, b_2, \ldots, b_9$ while the dimensions in the vertical direction are denoted by h_1, h_2, and h_3. The forcing inputs are six, u_1 and u_2 suspend the tractor, the cab and the

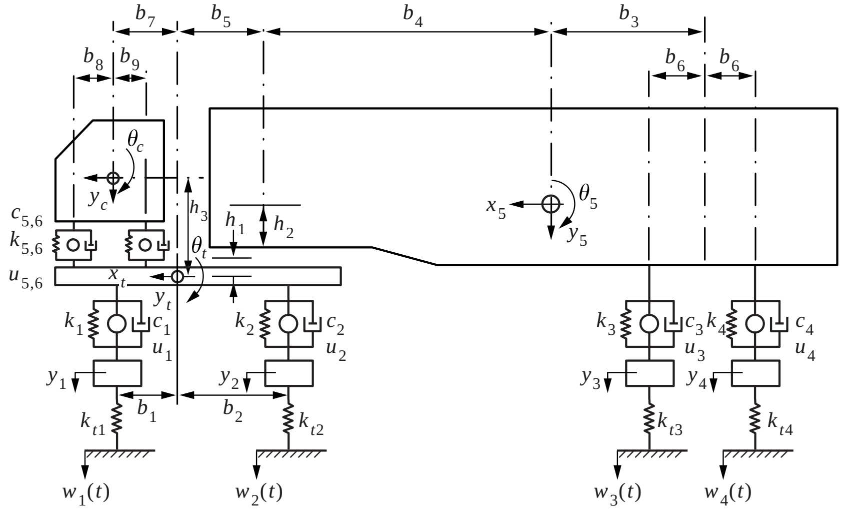

Figure 1. A 9-DOF tractor semitrailer vehicle model [17, 18].

semi-trailer, u_3 and u_4 suspend the semi-trailer and u_5 and u_6 suspend the cab. The four road inputs are w_1, w_2, w_3 and w_4.

3. SYSTEM REPRESENTATION

In this section the nine second-order differential equations are represented in a state space form which is more suitable for the problem. The state space representation leads to eighteen first-order differential equations assembled in matrix form as

$$\dot{x} = Ax + Bf + Dw, \tag{1}$$

where x is a state variable vector, f is a forcing input vector and w is a vector of road excitations imparted at the four wheels. These vectors are defined as follows:

$$x = [y_c, \theta_c, y_t, \theta_t, \theta_s, y_1, y_2, y_3, y_4, \dot{y}_c, \dot{\theta}_c, \dot{y}_t, \dot{\theta}_t, \dot{\theta}_s, \dot{y}_1, \dot{y}_2, \dot{y}_3, \dot{y}_4],$$

$$f = [u_1, u_2, u_3, u_4, u_5, u_6], \qquad w = [w_1, w_2, w_3, w_4]'.$$

The matrices A, B and D are all of constant coefficients resulting from the state space formulation. For the sake of convenience for the optimization problem of this study the vector of forcing inputs, f, is divided into two sub-vectors; the first one is $f = [ug]'$, where $u = [u_1, u_2, u_3, u_4]'$ and the second one is $g = [u_5, u_6]' = [g_1, g_2]'$. If one partitions the matrix B in equation (1) such that $B = [B_1, B_2]$, where B_1 is a sub-vector containing only the columns of B corresponding to the forcing input vector u, and B_2 is a sub-vector containing only the columns of B corresponding to the forcing input vector g, the state space form of equation (1) can be reproduced in the form

$$\dot{x} = Ax + B_1 u + B_2 g + Dw. \tag{2}$$

In the case of semi-active dampers for suspending the vehicle cab, the elements of the forcing input vector, g, can be extracted from the model dynamic equations as

$$g_1 = v_1(t)(+\dot{y}_c - b_8\dot{\theta}_c - \dot{y}_t + (b_7 + b_8)\dot{\theta}_t), \qquad g_2 = v_2(t)(+\dot{y}_c + b_9\dot{\theta}_c - \dot{y}_t + (b_7 - b_9)\dot{\theta}_t), \tag{3}$$

where $v_1(t)$ and $v_2(t)$ are rapidly changing damping coefficients in a range of predetermined values

$$v_{k\,\min} \leqslant v_k \leqslant v_{k\,\max}, \qquad k = 1, 2, \ldots, n_v, \tag{4}$$

where n_v is the number of (semi-active) rapidly switched dampers; equal to two in this study, and a condition of $0 \leqslant v_{k\,\min} < v_{k\,\max}$ must be satisfied. Note that $v_{k\,\min}$ should be greater than zero for the damper to be viscous. The forcing input vector g is then replaced by

$$g = (\text{diag } v)Vx, \tag{5}$$

where,

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -b_8 & -1 & +(b_7 + b_8) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & +b_9 & -1 & +(b_7 - b_9) & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\text{diag } v = \begin{bmatrix} v_1(t) & 0 \\ 0 & v_2(t) \end{bmatrix}.$$

From equations (2) and (5), it follows that

$$\dot{x} = Ax + B_1 u + B_2(\text{diag } v)Vx + Dw. \tag{6}$$

The last equation is a bilinear state equation because of the product of the state vector x and the semi-active control elements $v_1(t)$ and $v_2(t)$ in the third term of the right-hand side.

In the design of automobile suspensions the designer is often in a hard situation to decide how much preference is to be given to each performance measure. In fact, the most important measures of any vehicle suspension are the sprung mass accelerations taken in the bounce, pitch and roll directions, the suspension deflection which must be limited because of space limitations due to the modern compact designs of vehicles, and the tire deflection that plays a very vital role in the vehicle safety and handling. In order to meet the above performance qualities, the following performance index is chosen for the current problem, as presented in reference [19]:

$$J = E\Bigg[\alpha_1 a_{c1}^2 + \alpha_2 a_{c2}^2 + \alpha_3 \sum_{i=1}^{3} a_{ti}^2 + \alpha_4 \sum_{i=1}^{3} a_{si}^2 + \rho_1 \sum_{i=1}^{2} \Delta_{ci}^2 + \rho_2 \sum_{i=1}^{4} \Delta_{ti}^2 + \rho_3 \sum_{i=1}^{4} \Delta_{si}^2$$
$$+ \rho_4 + \sum_{i=1}^{4} \Delta_{wi}^2 + \sum_{i=1}^{4} \gamma_i u_i^2 + \sum_{i=1}^{2} \eta_i v_i^2\Bigg]. \tag{7}$$

Here E[.] denotes expected values or variances, a_{c1} is the cab vertical acceleration, a_{c2} is the cab pitch acceleration, a_{ti} is the accelerations obtained at three points, tractor centre of gravity and both ends of the tractor frame, a_{si} is the semi-trailer accelerations obtained at three points, semi-trailer centre of gravity and both ends of the semi-trailer frame, Δ_{ci} are cab suspension deflections, Δ_{ti} are tractor suspension deflections, Δ_{si} are semi-trailer suspension deflections, and Δ_{wi} are tyre deflections. Also, $\alpha_1 \cdots \alpha_4$, $\rho_1 \cdots \rho_4$, γ_i $(i = 1 \cdots 6)$, and η_i, $(i = 1, 2)$ are weighting factors which reflect designer's preferences.

The performance index of equation (7), after simple algebraic manipulation with the system equations of motion, can be recast in a more convenient form for the problem as follows:

$$J = \lim_{T\to\infty} \frac{1}{2T}\int_0^T [x'Q_1x + 2x'S_1u + u'R_1u + 2x'S_2(\text{diag } v)]Vx + x'V'(\text{diag } v)R_2(\text{diag } v)Vx$$
$$+ 2u'Q_{uv}(\text{diag } v)Vx + 2x'Q_5w + 2u'Q_3w + 2x'V'(\text{diag } v)Q_4w + w'Q_2w]\,\mathrm{d}t. \qquad (8)$$

This last performance index is a weighted sum of variances of state variables, x, active forcing (control) inputs, u, semi-active damping forces, diag $v.V$, and road excitation functions, w. It also minimizes the dynamic coupling between the state variables and (i) the control inputs, (ii) the semi-active inputs, and (iii) the road inputs. In addition, it minimizes the dynamic coupling between the active control inputs, u, and the semi-active control inputs, diag $v.V$, and also minimizes the dynamic coupling between both the active and semi-active control inputs and the road inputs. The matrices Q_1, Q_2, Q_3, Q_4, Q_5, Q_{uv}, R_1, R_2, S_1 and S_2 are all constant weighting matrices extracted by the manipulation of equation (7) and the model equations of motion. Since w appears here in the performance index, we assume instantaneous measurement or reconstruction of the road input. This will allow a feedforward part in the control input which reduces the current disturbances [15].

4. PREVIEW FULLY ACTIVE SUSPENSION

In this study a perfect preview of road irregularities is assumed. This requires a perfect sensing device to be installed in the front of the vehicle. The theory developed here is based on an assumption of sufficiently long preview time. A compromise is seeked by performing all the situations at a vehicle running speed of 15 m/s with preview time of 0·15 s. Basically, the theory requires the finite knowledge of road roughness ahead of the front wheel for $\sigma = [t, t + t_p]$, where $t_p \succ 0$ is the preview time, meaning that a sensor is supposed to measure a road roughness segment $l_p = Ut_p$, where U is the vehicle traveling speed. The sensed distance ahead of the vehicle is determined at the end by the overhang of the vehicle from the front axle. It is well known [10] that the pitch motion of both the tractor and the cab, θ_c and θ_t, should be compensated by keeping the absolute inclination of the sensor constant with respect to the road surface. This is quite important, since view angle variations change the previewed distance. This is one of the many reasons which have not allowed the use of an operational preview system.

THEOREM 1

Consider the optimization of the system described by equation (2) and the performance index of equation (8) with g replacing (diag v)Vx, and no explicit constraints of equation (4) are considered. If one introduces the notations

$$A_n = [A - B_1(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}(S'_1 - Q_{uv}R_2^{-1}S'_2)$$
$$- B_2(R_2 - Q'_{uv}R_1^{-1}Q_{uv})^{-1}(S'_2 - Q'_{uv}R_2^{-1}S'_1)],$$
$$Q_n = [Q_1 - S_1(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}(S'_1 - Q_{uv}R_2^{-1}S'_2)$$
$$- S_2(R_2 - Q'_{uv}R_1^{-1}Q_{uv})^{-1}(S'_2 - Q'_{uv}R_2^{-1}S'_1)],$$
$$Q_d = [D - B_1(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}(Q_3 - Q_{uv}R_2^{-1}Q_4)$$
$$- B_2(R_2 - Q'_{uv}R_1^{-1}Q_{uv})^{-1}(Q_4 - Q'_{uv}R_2^{-1}Q_3)],$$

$$Q_s = [Q_5 - S_1(R_1 - Q_{uw}R_2^{-1}Q'_{uw})^{-1}(Q_3 - Q_{uw}R_2^{-1}Q_4) - S_2(R_2 - Q'_{uw}R_1^{-1}Q_{uw})^{-1}(Q_4 - Q'_{uw}R_2^{-1}Q_3)],$$

$$R_n = [B_1(R_1 - Q_{uw}R_2^{-1}Q'_{uw})^{-1}(B'_1 - Q_{uw}R_2^{-1}B'_2) + B_2(R_2 - Q'_{uw}R_1^{-1}Q_{uw})^{-1}(B'_2 - Q'_{uw}R_2^{-1}B'_1)],$$

and if either (A_n, B_1) or (B_1, B_2) is a stabilizable pair [22] and $(A_n, Q_n^{1/2})$ is detectable [15], the control forces are then given by

$$u_o(t) = -(R_1 - Q_{uw}R_2^{-1}Q'_{uw})^{-1}[[(S'_1 - Q_{uw}R_2^{-1}S'_2) + (B'_1 - Q_{uw}R_2^{-1}B'_2)P_n]x(t) + (B'_1 - Q_{uw}R_2^{-1}B'_2)r_n(t) + (Q_3 - Q_{uw}R_2^{-1}Q_4)w(t)], \quad (9)$$

$$g_o(t) = -(R_2 - Q'_{uw}R_1^{-1}Q_{uw})^{-1}[[(S'_2 - Q'_{uw}R_2^{-1}S'_1) + (B'_2 - Q'_{uw}R_1^{-1}B'_1)P_n]x(t) + (B'_2 - Q'_{uw}R_1^{-1}B'_1)r_n(t) + (Q_4 - Q'_{uw}R_1^{-1}Q_3)w(t)], \quad (10)$$

where P_n is a non-negative definite solution of the algebraic Riccati equation

$$P_nA_n + A'_nP_n - P_nR_nP_n + Q_n = 0, \quad (11)$$

and the vector $r_n(t)$ is given by

$$r_n(t) = \int_0^{t_{prv}} + \tau_1 + \tau_2 + \tau_3 \, e^{A_{cn}\sigma}Qw \begin{bmatrix} O_1(t_{prv} - \sigma)w_1(t+\sigma) \\ O_2(t_{prv} + \tau_1 - \sigma)w_1(t+\sigma+\tau_1) \\ O_3(t_{prv} + \tau_1 + \tau_2 - \sigma)w_1(t+\sigma-\tau_1-\tau_2) \\ w_1(t+\sigma-\tau_1-\tau_2-\tau_3) \end{bmatrix} d\sigma, \quad (12)$$

where τ_1, τ_2 and τ_3 are time delays between road inputs taken between every two successive tires. O_1, O_2 and O_3 are Heaviside (unit step) functions such that; for example,

$$O_1(t_{prv} - \sigma) = \begin{cases} 1 & \text{for } \sigma \leqslant t_{prv}, \\ 0 & \text{for } \sigma \succ t_{prv}. \end{cases} \quad (13)$$

The proof of Theorem 1 is a minor modification of Case I of the proof of Theorem 2 in the Appendix when the inequality constrained are not considered. The closed loop matrix $A_{cn} = (A_n - R_nP_n)$ is to be asymptotically stable due to the non-negative definiteness of the Riccati solution in equation (11). Note here that the control forces of equations (9) and (10) consist of three main parts: (1) the first part is just like the one known in the LQ problem; (2) a preview part in which $r_n(t)$ appears, and which uses the future road information; (3) one which uses the current road measurements.

To this end, the terms "stabilize" and "detectable" have to be explained. In general, stabilizability means that the unstable modes in the control matrix A in equation (2) can be stabilized by the feedback action of the forcing inputs. Thus, the forcing inputs can derive the system from a given initial state to another state in a finite time. Detectability in the ability to determine the state of a system from certain output observations or measurements. Further reading about the stabilizability and the detectability of the linear and bilinear control systems can be found in reference [22].

5. PREVIEW HYBRID SUSPENSION

Now, one can return to the bilinear control system of equation (6). This system has independent additive and multiplicative control inputs. The additive control inputs, $u_1, \ldots, u_4$, are scalar and magnitude-unconstrained, but the multiplicative control inputs, v_1 and v_2, are scalar and magnitude-constrained. There are two shock (semi-active) absorbers of variable damping coefficients v_1 and v_2 for suspending the cab and four fully-active actuating (inputs) forces, $u_1, \ldots, u_4$, two of them for suspending the tractor and the other two are for suspending the semi-trailer. The damping coefficients v_1 and v_2 are to be continuously varied to ensure that the semi-active damping forces acts as closely as possible to fully active forces g_1 and g_2 given by equation (10). In other words, these optimum fully active forces g_1 and g_2 are to be replaced by semi-active damping forces of optimum time-varying coefficients v_1 and v_2 with limiting bounds as in equation (4). In all the damping states the fully active control forces are to be adapted all the time according to the updated value of the damping coefficients.

THEOREM 2

Consider the minimization of the bilinear control system described by equation (6) with respect to the performance criterion (8) under the system of inequality constraints (4). If one introduces the notation

$$
\begin{aligned}
A_m(v) &= [A - B_1(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}((S'_1 - Q_{uv}R_2^{-1}S'_2) - Q_{uv}R_2^{-1}(S'_2 - Q'_{uv}R_2^{-1}S'_1)) \\
&\quad + (B_2 - B_1R_1^{-1}Q_{uv})\,\mathrm{diag}\, v^*.V], \\
Q_m(v) &= [Q_1 - S_1(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}((S'_1 - Q_{uv}R_2^{-1}S'_2) - Q_{uv}R_2^{-1}(S'_2 - Q'_{uv}R_2^{-1}S'_1)) \\
&\quad + (S_2 - S_1R_1^{-1}Q_{uv})\,\mathrm{diag}\, v^*.V + V'\,\mathrm{diag}\, v^*(S'_2 - Q'_{uv}R_1^{-1}S'_1) \\
&\quad + V'\,\mathrm{diag}\, v^*(R_2 - Q'_{uv}R_1^{-1}Q_{uv})\,\mathrm{diag}\, v^*.V], \\
Q_r &= [D - B_1(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}((Q_3 - Q_{uv}R_2^{-1}Q_4) - Q_{uv}R_2^{-1}(Q_4 - Q'_{uv}R_2^{-1}Q_3))], \\
Q_u(v) &= [Q_5 - S_1(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}((Q_3 - Q_{uv}R_2^{-1}Q_4) - Q_{uv}R_2^{-1}(Q_4 - Q'_{uv}R_1^{-1}Q_3)) \\
&\quad + V'\,\mathrm{diag}\, v^*(Q_4 - Q'_{uv}R_1^{-1}Q_3)], \\
R_m &= [B_1(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}((B'_1 - Q_{uv}R_2^{-1}B'_2) + Q_{uv}R_2^{-1}(B'_2 - Q'_{uv}R_2^{-1}B'_1))],
\end{aligned}
$$

and if either $(A_m(v), B_1)$ or (B_1, B_2) is a stabilizable pair and $(A_m(v), Q_m^{1/2})$ is detectable, the fully and semi-active control forces are then given by

$$u_o^*(t) = u_o(v_k^*(t)), \tag{14}$$

$$
v_k^*(t) = \begin{cases} v_{k\,\min} & \text{if} \quad g_{ok}(t)x_k(t) \leqslant v_{k\,\min}x_k^2(t) \\ v_{k\,\max} & \text{if} \quad g_{ok}(t)x_k(t) \geqslant v_{k\,\max}x_k^2(t) \\ g_{ok}(t)/x_k(t) & \text{otherwise} \end{cases} \tag{15}
$$

for $k = 1, 2, \ldots, n_v$, where g_{oi} denotes components of the control vector g_o which is given by equation (10), x_k is the relative velocity variable across the kth semi-active damper, and $u_o(v_k^*(t))$ is the optimal fully-active force calculated as a function of the current damping coefficients of the semi-active dampers,

$$
\begin{aligned}
u_o(v_k^*(t)) = -(R_1 - Q_{uv}R_2^{-1}Q'_{uv})^{-1}[[(S'_1 - Q_{uv}R_2^{-1}S'_2) + (B'_1 - Q_{uv}R_2^{-1}B'_2)P_m]x(t) \\
+ (B'_1 - Q_{uv}R_2^{-1}B'_2)r_m(t) + (Q_3 - Q_{uv}R_2^{-1}Q_4)w(t)].
\end{aligned} \tag{16}
$$

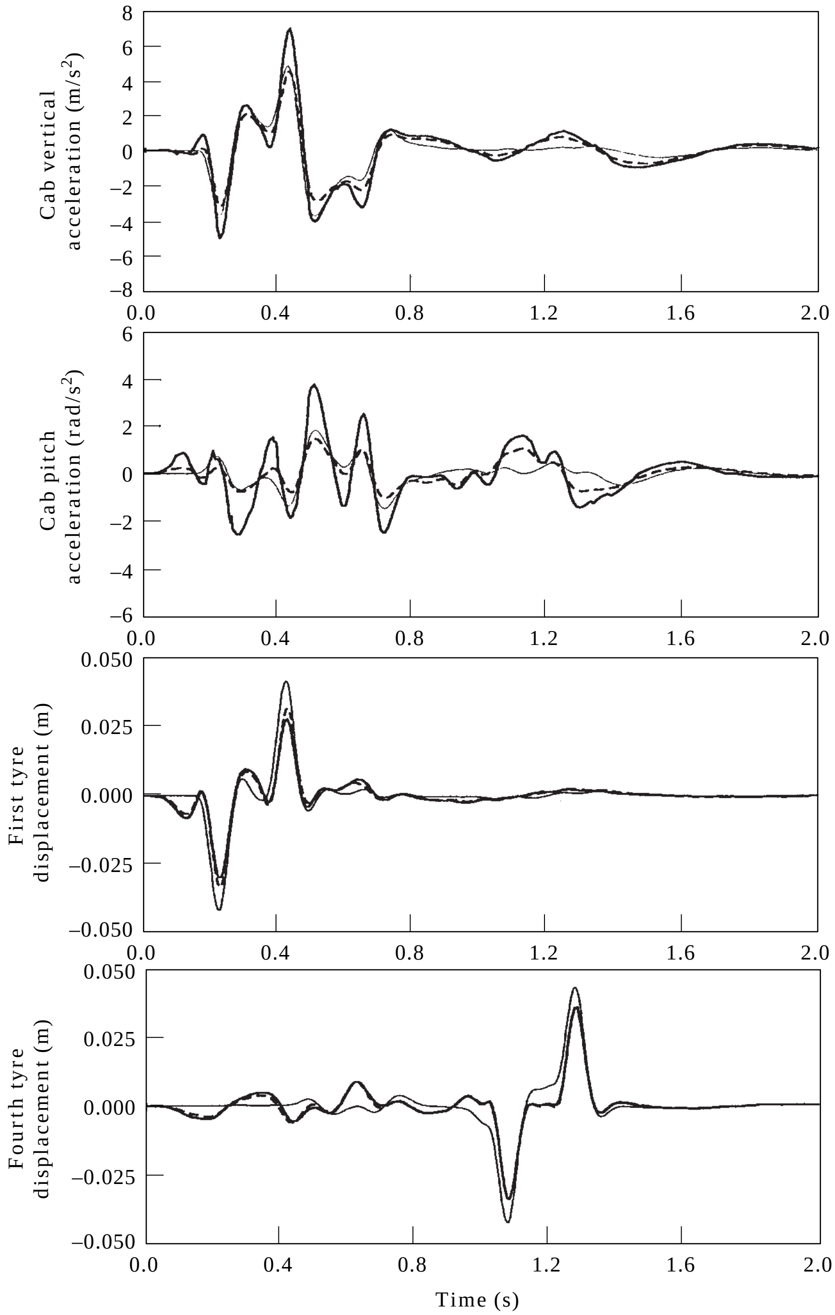

Figure 2. Time response of optimized suspension control systems at vehicle speed of 15 m/s and preview time of 0·15 s. ——, Active; — —, active with preview; ▬▬, hybrid with preview.

This requires the solution of the time-invariant matrix Riccati equation

$$P_m A_m(v) + A'_m(v) P_m - P_m R_m P_m + Q_m(v) = 0, \tag{17}$$

and the closed loop matrix, $A_{cm}(v) = (A_m(v) - R_m P_m)$, is to be asymptotically stable due to the non-negative definiteness of the Riccati solution in equation (17) and the positiveness of the semi-active damping coefficients. The calculation of the vector $r_m(t)$ is like the calculation of the vector $r_n(t)$ in equation (12) except that $A_{cm}(v)$ replaces A_{cn}. In the case when the last of the conditions of equation (15) is satisfied for all the constraints, the semi-active control force is just like the one generated by the fully active actuator given by equation (10). Otherwise, the semi-active damping force for each damper is set to one of its limiting values. Strictly speaking, for this study, one of the two dampers could be

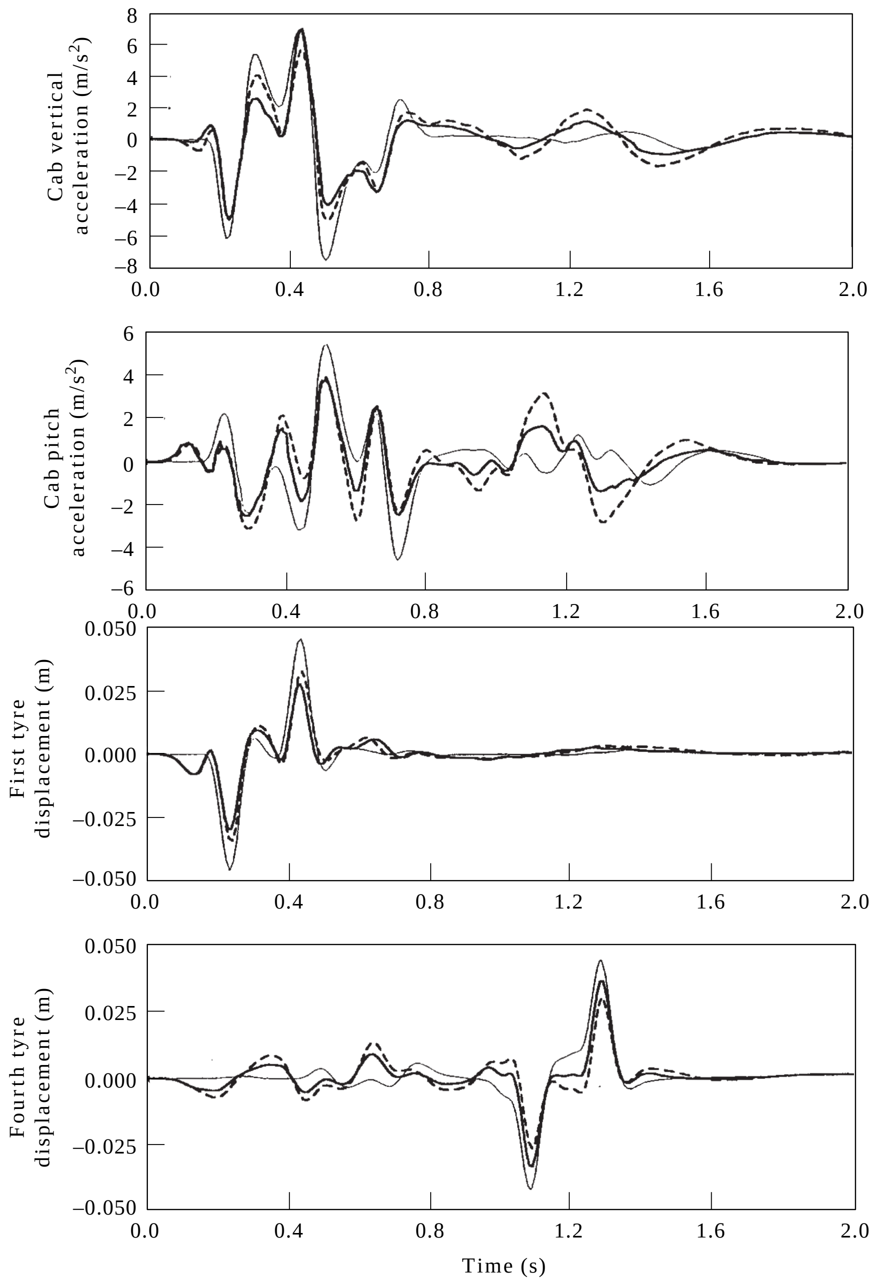

Figure 3. Time response of optimized suspension control systems at vehicle speed of 15 m/s and preview time of 0·15 s. ——, Semi-active; — —, semi-active with preview; ▬▬, , hybrid with preview.

TABLE 1

Comparison of various suspension designs in terms of the r.m.s. response measures

System	y_c (m/s^2)	θ_c (rad/s^2)	y_1 (m)	y_4 (m)
Passive	0·41	0·221	0·0038	0·0038
Active	0·21	0·07	0·0030	0·0035
Active with preview	0·31	0·099	0·0021	0·0034
Semi-active	0·37	0·15	0·0032	0·0032
Semi-active with preview	0·34	0·17	0·0021	0·0032
Hybrid with preview	0·33	0·12	0·0018	0·0034

set to either maximum or minimum limiting value while the other is satisfying the inequality constraints. The proof of Theorem 2 is given in the Appendix.

6. RESULTS AND DISCUSSIONS

A time-domain simulation process was performed by using the fifth-order Runge–Kutta method in order to explore the performance features of the control method presented in this paper relative to those of some other control methods. All the simulation results were obtained by considering ideal semi-active dampers. By an "ideal" damper is meant a viscous damper with a linear force–velocity curve and an instantaneous response in switching from one damping state to another. Of course, in practice, there are hardware limitations which affect the operation of semi-active dampers such as the off-state damping ratio, and the switching time required for the control valve to go from completely open to completely closed. The author in reference [23] showed that the off-state damping ratio should be $<0{\cdot}2$, and the response of the control valve should be less than approx. 0·014 s.

The vehicle response to a deterministic road input in the form of a hole followed by a bump is detected. The mathematical description of this type of road input is

$$w(t) = \begin{cases} -0{\cdot}025(1-\cos 20\pi(t-0{\cdot}15)) & \text{for} \quad t\in[0{\cdot}15, 0{\cdot}25] \\ +0{\cdot}025(1-\cos 20\pi(t-0{\cdot}15)) & \text{for} \quad t\in[0{\cdot}35, 0{\cdot}45] \\ 0 & \text{otherwise} \end{cases}$$

A preview time of 0·15 s at a vehicle travel speed of 15 m/s is considered. This means that if the overhung is 1·25 m the previewed distance ahead of the vehicle will be 1 m. The weighting parameters that appear in the performance index of equation (7) are taken as follows: $\alpha_1 = 5\times 10^2$, $\alpha_2 = 3\times 10^2$, $\alpha_3 = \alpha_4 = 0$, $\rho_1 = 1\times 10^5$, $\rho_2 = 1\times 10^5$, $\rho_3 = 1\times 10^5$, $\rho_2 = 1\times 10^7$, $\eta_1 = \eta_1 = 1\times 10^3$, and $\gamma_1=\gamma_2=\gamma_3=\gamma_4 = 1\times 10^{-5}$. The response measures

TABLE 2

Effect of the weighting parameters α_1 and α_2 on the operation of the hybrid controller

Weighting parameters α_1 and α_2	y_c (m/s^2)	θ_c (rad/s^2)	y_1 (m)	y_4 (m)
$\alpha_1 = 500$, $\alpha_2 = 300$	0·33	0·12	0·0018	0·0034
$\alpha_1 = 750$, $\alpha_2 = 350$	0·26	0·009	0·0027	0·0038

of the vehicle model are shown in Figure 2. In this figure the performance of the suggested controller versus fully active and fully active with preview is shown. Note here that the equations of the fully active controller can be directly extracted from equations (9) and (10) by setting $r_n(t) = 0$: i.e. no preview action is considered. Also, a comparison with semi-active and semi-active with preview suspensions is shown in Figure 3.

From Figure 2, it is obvious that the suggested (hybrid) control method provides less performance capabilities with respect to the cab and pitch accelerations. This agrees with the results obtained in reference [13] because short preview times like 0·15 s allow improvement in the tyre-to-road holding property rather than the ride quality. Thus the hybrid controller (see Figure 2) provides performance features even better than the active system with preview for the first tyre. It provides a good compromise for the designer if it is decided to reduce the overall cost by replacing broad-band actuators by rapidly switched dampers. It is worth noting that the designer can give preference to the cab isolation by using broad-band actuators for suspending the cab and rapidly switched dampers for suspending both the tractor and the trailer.

Figure 3 shows that the suggested hybrid control method provides better isolation capabilities than the semi-active system and semi-active active with preview system from both the ride quality and the tyre-to-road contact forces. This fact holds true except for the fourth tyre where the semi-active with preview system provides lower response peaks than the others. This is also clear from the r.m.s. values in Table 1. These r.m.s. values are obtained by the simulation of system response to a white noise road input for sufficiently long time. Note here that the results obtained here belong to the selected set of weighting parameters at the beginning of this section, meaning that the designer has the freedom to choose these parameters in order to match his own preferences. For example, from Table 2, changing the values of the weighting parameters α_1 and α_2 from 500 and 300 to 750 and 350, respectively, will lead to a noticeable improvement in the ride quality measures y_c and θ_c on the expense of worsening the tyre-to-road property which is indicated by y_1 and y_4.

Most importantly, the method presented in this paper is based on the assumption of full state measurements which is hard to meet in many applications. An estimation approach could be developed for knowing the state variables, which are hard to measure.

7. CONCLUSION

A preview control method for vibration suppression in mechanical systems has been presented. It is shown to have the feature of solving standard matrix equations in order to obtain closed form solutions. It is specifically developed for the optimization of systems having ideal broad-band actuators and (semi-active) rapidly switched dampers working altogether at the same time in one system. The optimal fully active forces are always obtained according to the damping state of the semi-active dampers. The application to a tractor semi-trailer vehicle has shown the effectiveness of the method as compared to that of others.

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APPENDIX: PROOF OF THEOREM 2

This proof is based on the many derivations by using calculus of variations as in references [14, 15, 21]. The statement of the optimization problem is to obtain the minimum value of the performance criterion of equation (8) subject to the system dynamic constraint of equation (6) and the explicit constraints of equation (4). Note here that each of the constraints (4) can be divided into two inequality constraints as follows:

$$v_k + v_{k\,\min} \leqslant 0, \qquad v_k - v_{k\max} \leqslant 0, \qquad k = 1, 2, \ldots, n_v. \tag{A1}$$

The Hamiltonian approach for such a problem is given by

$$H = 0{\cdot}5x'Q_1x + x'S_1u + 0{\cdot}5u'R_1u + x'S_2(\text{diag } v)Vx + 0{\cdot}5x'v'(\text{diag } v)R_2(\text{diag } v)Vx$$
$$+ u'Q_{uv}(\text{diag } v)Vx + x'Q_5w + u'Q_3w + x'V'(\text{diag } v)Q_4w + 0{\cdot}5w'Q_2w$$
$$+ \lambda'[Ax + B_1u + B_2(\text{diag } v)Vx + Dw] + l_k'(v_{\min} - v) + q_k'(v - v_{\max}), \tag{A2}$$

where the vector λ and the scalars l_k and q_k are Lagrange multipliers which weight the dynamic and inequality constraints, respectively. The necessary conditions for optimality are [21] $H_u = \partial H/\partial u = 0$, $H_v = \partial H/\partial v = 0$, $\partial H/\partial x = -\dot{\lambda}$, with $\lambda(T) = 0$, and either $H_{l_k} = 0$ or $H_{q_k} = 0$, $k = 1, 2, \ldots, n_v$ when the kth constraint of the set (A1) is active. The first three conditions lead to

$$\frac{\partial H}{\partial u} = 0 = S_1'x + R_1u + Q_{uv}(\text{diag } v)Vx + Q_3w + B_1'\lambda, \tag{A3}$$

$$\frac{\partial H}{\partial v} = 0 = \text{diag}\,(Vx)S_2'x + \text{diag}\,(Vx)R_2\,\text{diag}\,(Vx)v + \text{diag}\,(Vx)Q_{uv}u$$
$$+ \text{diag}\,(Vx)B_2'\lambda - l_k + q_k, \tag{A4}$$

$$\frac{\partial H}{\partial x} = -\dot{\lambda} = Q_1x + S_1u + S_2(\text{diag } v)Vx + V'\,(\text{diag } v)S_2'x + V'(\text{diag } v)R_2(\text{diag } v)Vx$$
$$+ V'(\text{diag } v)Q_{uv}'u + Q_5w + V'(\text{diag } v)Q_4w + A'\lambda + V'(\text{diag } v)B_2'\lambda. \tag{A5}$$

Equations (A3) and (A4) yield:

$$u = -(R_1 - Q_{uv}R_2^{-1}Q_{uv}')^{-1}[(S_1' - Q_{uv}R_2^{-1}S_2')x + (B_1' - Q_{uv}R_2^{-1}B_2')\lambda$$
$$+ (Q_3 - Q_{uv}R_2^{-1}Q_4)w + Q_{uv}R_2^{-1}\,\text{diag}\,(Vx)^{-1}(l_k - q_k)], \tag{A6}$$

$$\text{diag}\,(Vx)v = -(R_2 - Q_{uv}'R_1^{-1}Q_{uv})^{-1}[(S_2' - Q_{uv}'R_1^{-1}S_1')x + (B_2' - Q_{uv}'R_1^{-1}B_1')\lambda$$
$$+ (Q_4 - Q_{uv}'R_1^{-1}Q_3)w - \text{diag}\,(Vx)^{-1}(l_k - q_k)]. \tag{A7}$$

Substituting equations (A6) and (A7) into equations (6) and (A5) yields

$$\dot{x} = A_nx - R_n\lambda + Q_dw + Z_1\,\text{diag}\,(Vx)^{-1}(l_k - q_k), \qquad k = 1, 2, \ldots, n_v, \tag{A8}$$

$$\dot{\lambda} = Q_nx - A_n'\lambda + Q_sw - Z_2\,\text{diag}\,(Vx)^{-1}(l_k - q_k), \qquad k = 1, 2, \ldots, n_v, \tag{A9}$$

where the matrices A_n, R_n, Q_d, Q_s and Q_n are as defined in the text, and the matrices Z_1, and Z_2 are defined as follows:

$$Z_1 = -B_1(R_1 - Q_{uv}R_2^{-1}Q_{uv}')^{-1}Q_{uv}R_2^{-1} + B_2(R_2 - Q_{uv}'R_1^{-1}Q_{uv})^{-1},$$
$$Z_2 = -S_1(R_1 - Q_{uv}R_2^{-1}Q_{uv}')^{-1}Q_{uv}R_2^{-1} + S_2(R_2 - Q_{uv}'R_1^{-1}Q_{uv})^{-1} + V\,\text{diag } v.$$

CASE 1

None of the constraints (A1) is active. This is the case of constraints satisfaction in which it is convenient to write

$$l_k = q_k = 0, \qquad k = 1, 2, \ldots, n_v. \tag{A10}$$

Due to equation (A10), equations (A8) and (A9) become

$$\dot{x} = A_nx - R_n\lambda + Q_dw, \qquad \dot{\lambda} = -Q_nx - A_n'\lambda - Q_sw. \tag{A11}$$

One can introduce the vector $\lambda(t)$ in the form [15]

$$\lambda(t) = P_n(t)x(t) + r_n(t). \tag{A12}$$

Substituting equation (A12) into equations (A11) yields

$$-\dot{P}_n = P_n A_n + A_n' - P_n R_n P_n + Q_n, \qquad \dot{r}_n = (A_n' + P_n R_n) r_n - Q_w w, \tag{A13}$$

where $Q_w = P_n Q_d + Q_s$. If either (A_n, B_1) or (B_1, B_2) is a stabilize pair and $(A_n, Q_n^{1/2})$ is detectable, the solution of the Riccati equation (A13) tends to a constant matrix as in equation (11), and the vector $r_n(t)$ is then given by [15]

$$r_n(t) = \lim_{T\to\infty} \int_0^{T-t} \mathrm{e}^{A_{cn}\sigma} Q_w w(t+\sigma)\,\mathrm{d}\sigma. \tag{A14}$$

If the preview time is quite sufficient the last equation can be rewritten:

$$r_n(t) = \int_0^{t_{prv}} \mathrm{e}^{A_{cn}\sigma} Q_w w(t+\sigma)\,\mathrm{d}\sigma. \tag{A15}$$

In this case the control forces are given in equations (9) and (10), where diag $Vx.v$ replaces g_o in (10). This also completes the proof of Theorem 1.

CASE II

One of the constraints (A1) is active. For example, if the first constraint is active, it means that it can be set to either a minimum or maximum limiting value: i.e. $l_{1\neq 0}$ and $l_2 = l_3 = \cdots = l_{n_v} = 0$, or $q_{1\neq 0}$, $q_2 = q_3 = \cdots = q_{n_v} = 0$, $k = 1, 2, \ldots, n_v = 0$. Strictly speaking, either $H_{l_1} = 0$ and $v_1 = v_{1\min}$ or $H_{q_1} = 0$ and $v_1 = v_{1\max}$. Here, the optimum values of the forcing inputs are derived as functions of the new system structure. Also, the remaining inactive constraints of equation (A1) are to be determined from equation (10). From equation (A7), one gets

$$\begin{aligned} l_k - q_k = \operatorname{diag}(Vx)[(S_2' - Q_{uv}' R_1^{-1} S_1')x + (B_2' - Q_{uv}' R_1^{-1} B_1')\lambda \\ + (R_2 - Q_{uv}' R_1^{-1} Q_{uv})\operatorname{diag} v.V + (Q_4 - Q_{uv}' R_1^{-1} Q_3)w]. \end{aligned} \tag{A16}$$

If one considers the equalities

$$\begin{aligned} &(V' \operatorname{diag} v(B_2' - Q_{uv} R_1^{-1} B_1')) \\ &\qquad = (B_2 - B_1(R_1 - Q_{uv} R_2^{-1} Q_{uv}') Q_{uv} R_2^{-1} (R_2 - Q_{uv}' R_1^{-1} Q_{uv}) \operatorname{diag} v.V), \\ &S_1 R_1^{-1} Q_{uv} \operatorname{diag} v.V = S_1 (R_1 - Q_{uv} R_2^{-1} Q_{uv}')^{-1} Q_{uv} R_2^{-1} (R_2 - Q_{uv}' R_1^{-1} Q_{uv}) \operatorname{diag} vV, \end{aligned}$$

and substitutes equation (A16) into equations (A8) and (A9), one finds

$$\dot{x} = A_m(v)x - R_m\lambda + Q_r w, \qquad \dot{\lambda} = -Q_m(v)x - A_m'(v)\lambda - Q_u(v)w, \tag{A17}$$

where the matrices $A_m(v)$, R_m, Q_r, $Q_u(v)$, and $Q_m(v)$ are as defined in the text. The solution of the last two point boundary value problem can be sought by assuming that [15]

$$\lambda(t) = P_m(t)x(t) + r_m(t). \tag{A18}$$

Due to equation (A18), equations (A17) yield

$$-\dot{P}_m = P_m A_m(v) + A_m'(v) P_m - P_m R_m P_m + Q_m(v), \qquad \dot{r}_m = (A_m'(v) + P_m R_m) r_m - Q_g(v) w, \tag{A19}$$

where $Q_g(v) = P_m Q_r + Q_u(v)$. If either $(A_m(v), B_1)$ or $(A_m(v), B_2)$ is a stabilizable pair and $(A_m(v), Q_m^{1/2}(v))$ is detectable, the solution of the Riccati equation (A19) tends to a constant matrix as in equation (17), and the vector $r_m(t)$, for a sufficiently long preview time, can be calculated as

$$r_m(t) = \int_0^{t_{prv}} e^{A_{cm}(v)\sigma} Q_g(v) w(t+\sigma)\, d\sigma. \tag{A20}$$

The fully active control forces are then given by equation (16), and the components of semi-active damping coefficients that satisfy all the constraints can be calculated as follows:

$$\mathrm{diag}\,(\tilde{V}x)\tilde{v} = \tilde{g}_o. \tag{A21}$$

Note here that $\tilde{v}$ is a diagonal matrix having on its diagonal damping coefficients corresponding to the remaining inactive equality constraints and $\tilde{g}_o$ are their correspondents to be obtained from equation (10). In fact, Case II represents many cases of combinations of active and inactive equality constraints. Finally, the proof of sufficiency for the problem is lengthy but not difficult to be validated.